AP Calculus BC

Overview

Our AP Calculus BC class meets for a 42 minutes period 5 times a week. Students entering the class have taken a calculus honors class in their junior year which covered many pre-calculus prerequisites as well as an introduction to Calculus I topics. This course description and syllabus has been approved by the College Board.

Teaching Strategies/Philosophy

In my class, I promote a comfortable yet challenging learning atmosphere. I encourage students to question, explore and discover concepts as they are learning. The objective is to engage students in enjoyable lessons to promote a greater interest in mathematics. I want each student to realize they have something to contribute to the class and that others will benefit from their input as well as from my lessons. I want students to feel that sense of accomplishment after moving through a challenging topic and looking forward to what lies ahead of them. Students need to have a balanced approach to their solutions: Analytic, algebraic, numerical, graphic, and verbal methods of representing problems. Students must understand the proper use of technology with this course. Students must have a strong mathematics foundation and make the important connections between the concepts learned. The graphing calculator is a powerful tool to help develop a visual understanding of the material as well as an excellent computational resource to work on an application possibly in the area of science, business, or engineering.

Graphing Calculator Applications and Integration

We are fortunate in our school in that students enter BC Calculus with an extensive knowledge base in graphing calculator applications. Starting in the seventh grade, our students are presented with ways to integrate graphing calculator technology into their mathematical discovery and skill development. Functions such as window, trace, max/min, degree/radian, table, plot format, regression models, roots and intersections are fully developed and implemented. The calculator technology is further enhanced through the BC Calculus instruction.

Students learn how to visualize graphs (in rectangular, polar and parametric modes) as a way to interpret data and reach conclusions. While viewing the graph of f'(x), students clearly understand ways to extrapolate details about f(x) and f"(x). Through slope field and other differential functions calculator applications, students are able to visualize significant results based upon the output. Students are encouraged to explore alternate graphing calculator methods to find solutions and to develop a deeper understanding of the calculus. Through my instructional models, students quickly learn how to represent functions graphically, numerically, analytically and verbally. They also learn ways to integrate these representations into a meaningful cohesive entity.

Functions from Multiple Representations

As we all know, functions can be represented a variety to ways and methods. Certainly, these include graphical, numerical, analytical and verbal. As a regular and consistent component of my instructional model, my students develop an appreciation for the different function representations and, more importantly, learn how to integrate each of these forms to promote greater understanding of the calculus.

Each representation format is often associated with various advantages and disadvantages. For example, when a function is described in graphical form, my students learn ways to draw broad conclusions about the functions behavior and also how to reach specific details. Graphing calculator technology is very helpful in providing students with visual opportunities to connect multiple representations for a more meaningful learning experience. This type of instruction also allows students to practice better mathematical communication as they share ideas, discoveries and skills with their classmates.

Student Evaluation

Students are given homework on a regular basis. They work on questions assigned from the textbook and from supplemental sources. Towards the end of a topic, the students are assigned practice AP questions reflecting the different ideas learned. When review homework assignments, students share their methodology, calculator applications and problem-solving strategies.

In addition to homework, I assign three AP problems every two weeks to be collected. I rotate the calculator/non-calculator questions for each assignment. I encourage students to work cooperatively to discuss their ideas with their classmates but they are to turn in their own solutions. Students are assigned released multiple-choice from past AP exams on a monthly basis which is followed by a discussion in class regarding the examples from each test. Students are given quizzes to cover small pieces of material, usually work from the prior few days. Tests cover large amounts of material and are almost always cumulative. Tests are divided into calculator/non calculator sections and contain multiple-choice and free-response questions.

Primary Textbook

Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy.
Calculus: Graphical, Numerical, Algebraic. Needham, Mass.: Addison-Wesley, 2003.

Other Teaching Resources

Hughes-Hallet, Deborah, et al. *Calculus: Single Variable.* New York: John Wiley & Sons, 2005.

Larson, Ron, Bruce H. Edwards, and Robert P. Hostetler.
Calculus of a Single Variable. Boston: Houghton Mifflin, 2002.

Leithold, Louis. *The Calculus with Analytic Geometry*. New York: Harper & Row, Publishers, 1990.

Stewart, James
Calculus, Belmont, CA: Thomson Learning, Inc. 2003.

http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2118.html
http://www.sosmath.com/
http://archives.math.utk.edu/visual.calculus/
http://mathforum.org/library/drmath/sets/high_calculus.html
http://www.calculus.org/
http://www.calculus-help.com/funstuff/phobe.html
http://www.math.ucdavis.edu/~kouba/ProblemsList.html
http://www.plu.edu/~heathdj/java/
http://www.ma.iup.edu/projects/CalcDEMma/Summary.html
http://education.ti.com/educationportal/sites/US/homePage/index.html

Course Outline

I Limits & Continuity (8 days)
- a. Concept of a Limit
  - i. A graphic approach to limits
  - ii. Properties of limits
  - iii. Algebraic techniques to evaluating limits
  - iv. Limits involving infinity
- b. Continuity
  - i. Definition of Continuity
  - ii. Geometric understanding of the graphs of continuous functions
  - iii. Differentiability and Continuity

II The Derivative (8 days)
- a. Definition of the derivative – the tangent line problem
- b. Differentiation Rules and Procedures
- c. Derivatives of Trigonometric Functions
- d. Derivatives of Exponentials and Logarithmic Functions
- e. Derivatives of Inverses Trigonometric Functions
- f. Implicit Differentiation

Review and Assessment

III Applications of the Derivatives (16 days)
- a. Related Rates

b. Mean Value Theorem – Rolle's Theorem
c. Critical values, relative(local) and absolute (global)
d. The first and second derivative tests
e. Concavity and points of inflection
f. Comparing the graphs of *f, f' ,and f"*
g. Modeling Maximum and Minimum Problems
h. Local linearity & Newton's Approximation Method
i. Particle motion; position, velocity, and other rates of change including rectilinear motion

Review and Assessment

IV The Definite Integral (8 days)

a. Concept of Riemann Sums
    a. Includes left-hand, right-hand and midpoint applications
b. Estimation of change in a function using Riemann sums
c. Definition & Interpretation of the Definite Integral
d. Definite Integral & Antidifferentiation
e. The Fundamental Theorem of Calculus
f. Trapezoidal Rule
    a. Discovery exercise
g. Exercises to explore connection between concavity and over/under approximations

Review and Assessment

V Techniques of Integration (10 days)

a. Integration by u-substitution
b. Integration by parts
c. Integration by partial fractions

Quiz

VI Applications of Definite Integrals (18 days)

a. Area between two functions
    i. Discovery/Exploration activity (calculator active)
b. Volume of solids of revolution
    i. Disk method
    ii. Washer method
    iii. Shell method
    iv. Volume of solid with known parallel cross sectional areas
c. Arc length and surface of revolution

Review and Assessment

d. Definite integral as an accumulator
e. Work applications

f. Logistic Model

Quiz

VII Differential Equations (8 days)

a. Definition of a differential equation
b. Exponential growth & decay
c. Solving differential equations by separation of variables
d. Solving differential equations with initial conditions
e. Slope Fields
f. Euler's Method

Review and Assessment

VIII Parametric Functions (9 days)

a. Introduction to Parametric Equations
b. Derivatives of Parametric Equations & Length of Curve
    i. Derivative at a point
    ii. $\frac{d^2y}{dx^2}$
c. Vectors in the Plane
d. Vector Vales Functions
    i. Velocity, Speed, Acceleration, Direction, Distance

Review and Assessment

IX Polar Curves (8 days)

a. Polar coordinates and polar graphs
    i. Exploration activity
    ii. Polar graph lab experience (calculator active)
b. Derivatives of polar curves
c. Area in polar coordinates
d. Arc length of polar curves

Review and Assessment

X Improper Integrals (7 days)

a. L'Hopital's rule revisited
b. Related Rates of Growth
c. Improper Integrals
    i. Introduction to convergence tests
    ii. Discovery activity

Review and Assessment

XI Sequences & Series (26 days)

a. Sequences
    i. Review of properties of sequences

ii. Convergence of n^{th} term of a sequence

b. Series
   i. Introduction to Infinite Series and their properties
   ii. Convergent and Divergent Series (partial sums, telescoping, etc.)
   iii. Geometric Series
      a. Convergence of a Geometric Series
      b. Sum of geometric series
   iv. p-series
      a. Convergence of a p-series (illustrated by integral test)
      b. Harmonic Series and Alternating Harmonic Series
   v. Alternating Series
      a. Alternating series test
      b. Alternating series remainder/error
      c. Absolute and conditional convergence

d. Convergence Tests
   i. nth term test for divergence, integral test, direct comparison test, limit comparison test, alternating series test, ratio test, root test

e. Power Series
   i. Radius and Interval of Convergence
   ii. Testing Convergence at Endpoints
   iii. Term-by-term Differentiation and Integration
   iv. Representation of Functions by a Power Series

f. Taylor Series
   i. Taylor polynomials as approximations for functions
      1. Graphic exploration activity
   ii. Maclaurin polynomials
   iii. Taylor's Theorem with the Lagrange Error Form of the Remainder(Lagrange Error Bound)
   iv. Find a Taylor series for a function including *lnx*.
   v. Maclaurin series for sin*x*, cos*x*, e^x, $\frac{1}{1-x}$
   vi. Forming a new series from a known Taylor series

Review and Assessments

Review for the AP Calculus BC Test (18 days)

During this period, students work on free-response questions from the released exams. Students are given two days to complete six questions. The questions are collected and graded according to the scoring guidelines and then the questions and scoring are discussed the next day in class. Students are given the opportunity to experience rubric scoring and the AP scoring process. Over the course of the year, students learn to communicate in a mathematical fashion and to provide all necessary justifications.

Similarly, students make extensive use of graphing calculator technology (TI83+ or TI84+) as it is incorporated into every unit of study.

Students are also completing multiple choice questions on a weekly basis. These questions are addressed in class as well. Students are encouraged to work cooperatively on in-class assignments and extended problem sets. Several assessments will be administered during class meeting times that are similar in design to the questions being studied by the students. Special consideration is given to assessment items that represent multiple topics. This ensures mastery of all the necessary concepts reflected in the released AP examinations and further develops an appreciation for the integrated nature of mathematics.